P G DEPARTMENT OF MATEMATICS

Syllabus for the course of M.Phil/Ph.D
Session 2010-11

Note:- The question paper shall comprise of two parts :

Part (I) Subjective part containing Six questions of which Four questions must be attempted (40 Marks);

Part (II) Multiple choice objective part containing 30 questions and all are to be attempted (30 Marks).

Unit I (Advanced Abstract Algebra)

Cyclic groups, Cauchy's and Sylow's theorems, direct product, sovable groups, Euclidian rings, polynomial rings, irreducibility criteria, prime fields, extension of fields, splitting field, elements of Galois Theory, Problems of Antiquity.

Unit II (Real Analysis & Lebesgue Theory)

Infinite series, uniform convergence, Riemann-Stieltjes integration, convergence of improper integrals, inequalities, functions of several variables, Taylor's theorem, inverse and implicit function theorems, Lebesgue measure and its properties, Lebesgue Measurable and Borel sets, measurable functions and their characterization, Lebsgue integral and its properties, convergence theorems involving Lebesgue integration, Fundamental Theorem of Calculus for Lebesgue Integration, Absolute continuity and bounded variation, Stienhaus's Theorem on sets of positive measure, Ostrovski's theorem on measurable solutions of Cauchy's functional equation.

Unit III (Complex Analysis)

Analytic functions, Complex Integration, Cauchy's Theorem and its converce, Cauchy's Integral formula, Liouville's Theorem and its applications, Taylor's and Laurent's Theorems, classification of singularities and their characterization, Casorati-Weierstrass Theorem, Cauchy Residue Theorem, Contour Integration, Linear Transformation, their properties and classifications, Conformal mappings, Maximum Modulus Theorem, Schwarz's Lemma, Argument Principle, Rouche's Theorem, Poisson Integral formula, Poisson Jenson formula, Hadamard's Three Circle Theorem, Borel Caratheodory Theorem, Power series, Analytic continuation.

Unit IV (Differential Equations & Theory of Numbers)

Singular solutions, Initial Vale problems of 1^{st} order ODE, Picard's Theorem, Existence and Uniqueness of solution, Solution in series, Simultaneous equation $\frac{dx}{P}=\frac{dy}{Q}=\frac{\partial z}{R}$ and its solution, Total Equation Pdx+Qdy+Rdz = 0 and its solution, Existence of solution of PDE's of first order, Lagrange's and Charpit's methods, Classification and solution of second and higher order PDE's with constant coefficient.
Divisibility, Prime numbers, Fundamental Theorem of Arithmetic, Radix Diophantine presentation, Linear Equation, Congruences, CRS, RRS and their properties, Fermat's Euler's Theorem with applications, Euler's ϕ functions, Wilson's Theorem, Quadratic residues.

Unit V (Matrix Theory & Differential Geometry)

Characteristic Equation of a matrix, Caley-Hamilton Theorem, Rank of a matrix, Solution of homogeneous and non homogeneous equations, Eigenvalues and Eigen-vectors, orthogonal reduction of real matrices, normal matrices, quadratic forms, Kronecker-Lagrange's reduction, rank, index and signature, parametrization of curves, plane and space curves, curvature and torsion, Ferenet-Serret formula, Helices and their characterization, involutes and evolutes, Wiengarten equation, Codazzi-Manardi equations, geodesics, curvilinear coordinates, fundamental magnitudes, Meunsier's theorem, Euler's theorem, Rodrigue's formula, Fourier series.

Unit VI (Topology & Functional Analysis)

Completion of a metric space, convergence and completeness, Baire's category theorem, Cantor's intersection theorem, continuity and uniform continuity, Banach's contraction principle and simple applications, topological spaces, countability and separation axioms, product topology, compactness, boundedness and total boundedness in metric spaces, Urysohn's Lemma, Tietze's extension theorem, Banach spaces, continuous linear operators and their characterization, Finite dimensional Banach space, equivalence of norms,, Dual spaces of classical Banach spaces, Hahn-Banach theorem and its applications, uniform boundness principle, open mapping theorem and closed graph Theorem, separable Banach spaces, Hilbert spaces, Cauchy-Schwarz inequality, Parallelogram law, orthonormal systems(o.n), Bessel's inequality, Parseval's identity for complete o.n. systems, Riesz-representation theorem, spectral theorem for normal operators on finite-dimensional Banach spaces.

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