

**Ph.D. ENTRANCE TEST-2023****SUBJECT (MATHEMATICS)**

Total Questions: 100

Time Allowed : 110 Minutes

Roll No.

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**Instructions for Candidates**

1. Write your roll number in the space provided at the top of this page of question booklet and fill up the necessary information in the spaces provided on OMR Answer sheet.
2. OMR Answer sheet has an original copy and a candidate's copy glued beneath it at the top. While making entries in the original copy, candidate should ensure that the two copies are aligned properly so that the entries made in the original copy against each item are exactly copied in the candidate's copy.
3. All entries in the OMR answers sheet including answers to questions are to be recorded in the original copy only.
4. Use only blue/ black ball point pen to darken the circle of correct / most appropriate response. In gel/ ink pen or pencil should be used.
5. **Do not darken more the one circle of option for any question. A question with more than one darkened response shall be considered wrong.**
6. **There will be no "Negative Marking" for wrong answers.**
7. Only those candidates who would obtain positive score in entrance test examination shall be eligible for admission
8. Do not make any stray mark on the OMR sheet
9. Calculators and mobiles shall not be permitted inside the examination hall
10. Rough work, if any, should be done on the blank sheets provided with the question booklet.
11. OMR answer sheet must be handled carefully and it should not be folded or mutilated in such case it will not be evaluated.
12. Ensure that your OMR Answer sheet has been signed by the invigilator and the candidate himself/herself.
13. At the end of the examination hand over the OMR answer sheet to the invigilator who will first tear off the original OMR sheet in presence of the candidate and hand over the candidate's copy to the candidate.
14. If any of the information in the response sheet/question paper has been found missing or not mentioned as stated above the candidate is solely responsible for that lapse.

**Part I (General Aptitude 2023)**

1. Tariq wants to sell a watch at a profit of 20%. He bought it at 10% less and sold it at ₹ 30 less, but still he gained 20%. The cost price of watch is.....
  - A. ₹ 250
  - B. ₹ 225
  - C. ₹ 240
  - D. ₹ 220
2. If today is Sunday then three days from now will be.....
  - A. Saturday
  - B. Friday
  - C. Thursday
  - D. Wednesday
3. Absar is brother of Mehdi. Iqra is sister of Gulshan. Mehdi is son of Iqra. How is Absar related to Iqra?
  - A. Son
  - B. Brother
  - C. Nephew
  - D. Father
4. Ankit can do a piece of work in 6 days and Basharat in 9 days. How many days will both take together to complete the work?
  - A. 7.5 days
  - B. 5.4 days
  - C. 3.6 days
  - D. 3 days
5. The book "To Hell and Back: Humans of COVID" is authored by?
  - A. Kavitha Iyer
  - B. Jhumpa Lahiri
  - C. Barkha Dutt
  - D. Arundhati Roy
6. If PARTICLE is coded RCTVKENG, then how is SCIENCE coded?
  - A. TBJUOMF
  - B. TDJFODF
  - C. UEKGPEG
  - D. QBSUDMF
7. Where is the headquarter of the United Nations Environment Programme (UNEP) located?
  - A. Nairobi, Kenya
  - B. Venice, Italy
  - C. Munich, Germany
  - D. Geneva, Switzerland
8. Two years ago, Jane's age was three times Sam's age. If Jane is now 18, how old is Sam?
  - A. 6 years
  - B. 8 years
  - C. 10 years
  - D. 12 years
9. If WORK is coded as 4-12-9-16, then how will WOMAN be coded?
  - A. 4-12-14-26-13
  - B. 4-26-14-13-12
  - C. 23-12-26-14-13
  - D. 123-15-13-1-14
10. Which of the following states is not included in the sixth schedule of Indian Constitution?
  - A. Meghalaya
  - B. Tripura
  - C. Mizoram
  - D. Manipur

11. Letter : Word

- A. Homework : School
- B. Club : People
- C. Product : Factory
- D. Page : Book

12. The speed of a bus is 54 km/h if we don't let it stop at any point. If the bus stops at the bus-stops, the speed of the bus is 45 km/h. What is the time that the bus stops for per hour?

- A. 7 mins
- B. 10 mins
- C. 21 mins
- D. 22 mins

13. Blood does not coagulate inside the body due to the presence of \_\_\_\_\_?

- A. Fibrin
- B. Haemoglobin
- C. Heparin
- D. Plasma

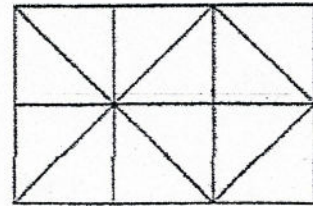
14. If a triangle has angles measuring 30 degrees, 60 degrees, and 90 degrees, what type of triangle is it?

- A. Equilateral
- B. Isosceles
- C. Scalene
- D. Right-angled

15. The ..... of the Minister's statement cannot be verified by people who have no access to official records.

- A. veracity
- B. verbosity
- C. ambiguity
- D. validity

16. The number of squares in the given figure is.....



- A. 7
- B. 8
- C. 9
- D. 10

17. What is the percentage of profit if the cost price is 95% of the selling price?

- A. 5%
- B. 5.26%
- C. 4%
- D. 4.75%

18. If you start facing east and turn 135 degrees clockwise, which direction are you facing now?

- A. North
- B. West
- C. North-East
- D. South-East

19. Pradhan Mantri Garib Kalyan Anna Yojana (PMGKAY) has been extended till which year recently?

- A. 2025
- B. 2028
- C. 2030
- D. 2032

20. Who is the present chairman of ISRO?

- A. Sh. Heeralal Samariya
- B. Sh. Harsh Chouhan
- C. Sh. Ravneet Kaur
- D. Sh. S Somanath

Ph.D. Entrance Examination (2023)

Subject: Mathematics

Section B

21. Which of the following is true.

- (A) Every solvable group is of prime power order
- (B)  $A_4$  has a subgroup of order six
- (C)  $S_3$  is decomposable
- (D) Group of order fifteen is abelian

22. Which of the following is not true.

- (A)  $S_3$  is a homomorphic image of  $S_4$
- (B) Group of order 56 is simple
- (C) Group of order 159 is cyclic
- (D) Group of even order has an element of order two

23. Which of the following is true.

- (A) A cube of twice the volume of the unit circle is constructible by ruler and compass
- (B)  $\mathbb{Z}[\sqrt{-5}]$  is UFD
- (C) Every irreducible element of a PID is prime
- (D) None of these

24. What is the number of irreducible polynomials of degree four in  $\mathbb{Z}_2$ .

- (A) 1
- (B) 2
- (C) 3
- (D) None of these

25. What is the order of the intersection of any two distinct Sylow-2-subgroups of the group of order 48.

- (A) 4
- (B) 12
- (C) 16
- (D) None of these

26. If the measure of a countable set is  $\alpha$ , then  $3\alpha + 5$  equals

- (A) 8
- (B) 11
- (C) 2
- (D) None of these

27. If  $2 \int_0^1 \frac{\log x}{\sqrt{1-x^2}} = A \pi$ , then  $A$  equals.

- (A)  $\log 2$
- (B)  $-\log 2$
- (C)  $\frac{1}{2} \log 2$
- (D) None of these

28. If  $k$  is the outer measure of the generalized Cantor set  $C(\alpha)$ ,  $0 < \alpha < 1$ , then  $k + \alpha$  equals

- (A)  $1 - \alpha$
- (B)  $2 - \alpha$
- (C)  $\alpha - 1$
- (D) None of these

29. Which of the following real valued function on  $(0, 1)$  is uniformly continuous.

- (A)  $f(x) = \sin\left(\frac{1}{x}\right)$
- (B)  $f(x) = \frac{\cos x}{x}$
- (C)  $f(x) = \frac{1}{x}$
- (D) None of these

30. What is the value of  $\int_C \frac{dz}{z-2}$  around the square with vertices at  $2 \pm 2i$ ,  $-2 \pm 2i$ .

- (A)  $\pi i$
- (B)  $-\pi i$
- (C)  $2\pi i$
- (D) None of these

31. What is the number of roots of the equation  $8z^4 - 6z + 5 = 0$ ,  $z$  complex, in the fourth quadrant.

- (A) 3
- (B) 2
- (C) 1
- (D) None of these

32. What is the residue of the function  $f(z) = \frac{\cot z \coth z}{z^3}$  at  $z = 0$ .

- (A)  $\frac{7}{45}$
- (B)  $-\frac{7}{45}$
- (C)  $\frac{8}{45}$
- (D) None of these

33. Which of the following is true.

- (A) The function  $f(z) = \frac{e^z}{z^2}$  is meromorphic in  $\mathbb{C}$
- (B) The function  $f(z) = |z^2|$  is analytic at any point
- (C)  $\tan z$  is entire function
- (D) None of these

34. What is the partial differential equation formed by eliminating the arbitrary function from  $z = x + y + f(xy)$ .

- (A)  $p = q$
- (B)  $py = q - x$
- (C)  $px - qy = x - y$
- (D) None of these

35. A solution curve of the differential equation  $x \frac{dy}{dx} = 2y$  passing through  $(1, 2)$ , also passes through

- (A)  $(2, 1)$
- (B)  $(0, 0)$
- (C)  $(-1, 1)$
- (D) None of these

36. Let  $\tau(n)$  denote the number of divisors of  $n$ . If  $\tau(18) = x$  and  $\tau(19) = y$ , then  $\tau(xy) \tau(x + y)$  equals

- (A) 12
- (B) 16
- (C) 18
- (D) None of these

37. If 95 can be written as  $7^2x + 7y + z$ , then  $x + y + z$  equals

- (A) 11
- (B) 12
- (C) 15
- (D) None of these

38. If  $\alpha, \beta, \gamma$ , ( $\alpha < \beta < \gamma$ ), are the eigenvalues of the matrix  $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ ,

then  $2\beta + 3\alpha + \gamma$  equals

- (A) 18
- (B) 12
- (C) 6
- (D) None of these

39. Which of the following system of vectors is linearly dependent.

- (A)  $(\frac{1}{4}, 0, -\frac{1}{4})$ ,  $(\frac{1}{3}, -\frac{1}{3}, 0)$ ,  $(0, \frac{1}{2}, -\frac{1}{2})$
- (B)  $(1, 9, 9, 8)$ ,  $(2, 0, 0, 8)$ ,  $(2, 0, 0, 3)$
- (C)  $(-4, 2)$ ,  $(9, 1)$ ,  $(5, 3)$
- (D) None of these

40. The spherical indicatrix of a curve is a circle if and only if the curve is a

- (A) line
- (B) circle
- (C) cylinder
- (D) None of these

41. What is the locus of the mid points of the chords of a circular helix.

- (A) paraboloid
- (B) right helicoid
- (C) cone
- (D) None of these

42. Which of the following is true.

- (A) Any union of closed sets is closed
- (B) A subspace of a separable space is always separable
- (C) Every open sphere is an open set
- (D) None of these

43. If  $d(a, b)$  is a metric on  $X$ , then which of the following is not a metric on  $X$ .

- (A)  $\sqrt{d(a, b)}$
- (B)  $\frac{d(a, b)}{1 + d(a, b)}$
- (C)  $\min\{1, d(a, b)\}$
- (D)  $\max\{1, d(a, b)\}$

44. For  $x = (x_1, x_2, x_3, \dots) \in l_1$ , let  $A(x) = (0, x_1, 2x_2, x_3, 2x_4, x_5, 2x_6, \dots)$ . Then  $\lim_{n \rightarrow \infty} \|A^n\|^{\frac{1}{n}}$  equals

- (A) 1
- (B) 2
- (C)  $\sqrt{2}$
- (D) None of these

45. What is the orthogonal dimension of a Hilbert space having no complete orthonormal sets.

- (A) 0
- (B) 1
- (C)  $\infty$
- (D) None of these

46. A separable infinite dimensional complex Hilbert space is isometrically isomorphic to

- (A)  $c_0$
- (B)  $l_2$
- (C)  $l_3$
- (D) None of these

47. Which of the following is a degree sequence of a simple graph.

- (A) [1, 1, 2, 3]
- (B) [1, 1, 3, 3]
- (C) [0, 1, 2, 3]
- (D) None of these

48. Let  $x$  and  $y$  be respectively the order of the graphs  $C_4 \vee K_5$  and  $K_{3,4}$ . Then  $x - 3y + 2$  equals

- (A) 34
- (B) -16
- (C) 15
- (D) None of these

49. Let  $x$  and  $y$  be respectively the number of pendant vertices in the binary tree with order 25 and 13. Then  $2x - 3y$  equals

- (A) 13
- (B) 12
- (C) 5
- (D) None of these

50. What is the sum of the entries of the incidence matrix of the graph  $K_2 \vee K_2$ .

- (A) 16
- (B) 12
- (C) 8
- (D) None of these

## Section C

51. Which of the following statements is/are true.

- (1) The order of a Sylow 3-subgroup of a group of order 54 is 27.
- (2) A group of order 108 is simple.
- (A) both (1) and (2)
- (B) only (1)
- (C) only (2)
- (D) Neither (1) nor (2)

52. If  $x$  and  $y$  are respectively the number of non-abelian groups of order 39 and 165, then  $7x + 11y$  equals

- (A) 18
- (B) 16
- (C) 14
- (D) None of these

53. Which of the following statements is/are true.

- (1) Every field contains a prime field as a subfield
- (2) Every ideal of the polynomial ring  $F[x]$  is principal
- (A) both (1) and (2)
- (B) only (1)
- (C) only (2)
- (D) Neither (1) nor (2)

54. Which of the following statements is/are true

- (1) Product of primitive polynomials is primitive
- (2) The characteristic of an integral domain is  $3k$ ,  $k \geq 2$
- (A) both (1) and (2)
- (B) only (1)
- (C) only (2)
- (D) Neither (1) nor (2)

55. Which of the following statements is/are true

- (1)  $\sqrt{2} + \sqrt[3]{5}$  is algebraic over  $\mathbb{Q}$  of degree 5
- (2) An extension  $E$  of degree 2 over a field  $F$  is always a normal extension of  $F$
- (A) both (1) and (2)
- (B) only (1)
- (C) only (2)
- (D) Neither (1) nor (2)

56. Which of the following rings is/are PID.

- (1)  $\mathbb{R}[X, Y]/\langle X^2 + 1, Y \rangle$
- (2)  $\mathbb{Z}[X]/\langle X^2 + 1 \rangle$
- (A) both (1) and (2)
- (B) only (1)
- (C) only (2)
- (D) Neither (1) nor (2)

57. Which of the following statements is/are true.

- (1) The ring  $R = \mathbb{C}[x]/(x^2 + 1)$  has exactly two prime ideals
- (2) Every Galois group is simple
- (A) Both (1) and (2)
- (B) (1) only
- (C) (2) only
- (D) Neither (1) nor (2)

58. Which of the following series is convergent.

- (A)  $\sum_{n=1}^{\infty} \frac{\log n}{n}$
- (B)  $\sum_{n=2}^{\infty} \frac{1}{n \log n}$
- (C)  $\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right)$
- (D) None of these

59. Which of the following statements is/are true.

- (1) A singleton set is of measure zero
- (2) A step function is simple
- (A) Both (1) and (2)
- (B) (1) only
- (C) (2) only
- (D) Neither (1) nor (2)

60. If  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left( \frac{1}{\sqrt{2} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{6}} + \cdots + \frac{1}{\sqrt{2n} + \sqrt{2n+2}} \right) = a$ , then  $a^2 + a\sqrt{2} - 1$  equals

- (A)  $\frac{1}{3}$
- (B)  $\frac{1}{2}$
- (C)  $\frac{1}{2}$
- (D) None of these

61. Which of the following is/are true.

- (1)  $f(x, y) = \sqrt{|xy|}$  is differentiable at the origin
- (2) The function  $f(x) = |\log x|$  is differentiable at  $x = 1$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

62. Which of the following statements is/are true.

- (1) Every Borel set is Lebesgue measurable
- (2) Every open set and closed set is Lebesgue measurable
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

63. Which of the following integrals is convergent.

- (A)  $\int_0^\pi \frac{\sqrt{x}}{\sin x} dx$
- (B)  $\int_1^2 \frac{\sqrt{x}}{\log x} dx$
- (C)  $\int_0^1 \frac{\log x}{\sqrt{x}} dx$
- (D) None of these

64. Which of the following statements is/are true.

- (1) Lebesgue outer measure  $m^*$  is translation invariant
- (2) Collection of Lebesgue measurable sets is  $\sigma$ -algebra
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

65. If  $\int_0^\infty \frac{x}{\sinh x} dx = a$  and  $\int_0^\infty \frac{\sin x}{x} dx = b$ , then  $4a + 2b$  equals

- (A)  $2\pi + 1$
- (B)  $\pi + 1$
- (C)  $\pi + 2$
- (D) None of these

66. Which of the following statements is/are true.

(1) If  $f(z)$  is an entire function, then the Taylor series of  $f(z)$  is convergent for all  $z$

(2) An analytic function is bounded in the neighborhood of an isolated singular point

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

67. What is the angle of rotation at the point  $z = \frac{1+i}{2}$  under the mapping  $w = z^2$ .

(A)  $\frac{2\pi}{3}$

(B)  $\frac{\pi}{2}$

(C)  $\pi$

(D) None of these

68. If  $a$  is the residue of  $f(z) = \frac{\operatorname{cosec} z \operatorname{cosech} z}{z^3}$  at  $z = 0$  and  $b$  is the residue of  $f(z) = \frac{\sin z}{z^2}$  at  $z = 0$ , then  $3b + 60a$  equals

(A) 4

(B) 3

(C) 2

(D) None of these

69. Which of the following statements is/are true.

(1) The mobious transform takes circle into square

(2) A branch of logarithm function is analytic function

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

70. Which of the following statements is/are true.

(1) If  $h$  and  $\lambda$  are respectively the genus and order of an entire function, then  $h \leq \lambda \leq h + 1$  in the second quadrant is one

(2) The number of bilinear transformations whose fixed points are the roots of the equation  $x^2 + 1 = 0$  is two

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

71. Which of the following statements is/are true.

- (1) The function  $\frac{1}{2}\log(x^2 + y^2)$  is harmonic
- (2) The function  $e^x(\cos x + i\sin y)$  is holomorphic
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

72. What is the singular solution of the differential equation  $3p^2e^y - px + 1 = 0$ .

- (A)  $3x - 4e^y = 0$
- (B)  $x^2 - y^2 = 0$
- (C)  $x^2 - 12e^y = 0$
- (D) None of these

73. What is the general solution of the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0.$$

- (A)  $z = \phi_1(y) + \phi_2(y + x) + \phi_3(y + 2x)$
- (B)  $z = \phi_1(x) + \phi_2(y - x) + \phi_3(2y + x)$
- (C)  $z = \phi_1(y) + \phi_2(y - x) + \phi_3(y - 2x)$
- (D) None of these

74. What is the solution of the differential equation

$$2x + y^2 + 2xz \left( \frac{dx}{dt} \right) + 2xy \left( \frac{dy}{dt} \right) + x^2 \left( \frac{dz}{dt} \right) = 1.$$

- (A)  $x + y^2 + zxy + t = e$
- (B)  $x^2 + xy^2 + zx^2 - t = e$
- (C)  $x^2 + y + zy^2 - t = e$
- (D) None of these

75. Which of the following statements is/are true.

- (1) 12 divides  $F_n - 5$  for all  $n \geq 1$
- (2)  $n^3 - n$  is divisible by 4,  $n \geq 2$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

76. Which of the following statements is/are true.

- (1) For prime  $p$ , the number of roots of the congruence  $x^{p-1} \equiv 0 \pmod{p}$  is  $p - 2$
- (2) For  $n \geq 1$ ,  $\sum_{d|n} \phi(d) = n - 1$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

77. Which of the following is the solution of the congruence

$$f(x) = x^3 + 2x^2 + 3x + 1 \equiv 0 \pmod{5}.$$

- (A)  $x \equiv 1 \pmod{5}$
- (B)  $x \equiv 2 \pmod{5}$
- (C)  $x \equiv 3 \pmod{5}$
- (D) None of these

78. Which of the following is/are true.

- (1) Greatest common divisor of any two Fermat numbers is a multiple of 3
- (2) For positive integers  $a$  and  $b$ ,  $(a, b)[a, b] = ab + 1$ , where  $(a, b)$  is greatest common divisor and  $[a, b]$  is least common multiple of  $a$  and  $b$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

79. Which of the following is/are true.

- (1) A square matrix is unitarily similar to a triangular matrix
- (2) A non-zero nilpotent matrix is similar to a diagonal matrix
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

80. Which of the following statements is/are true.

- (1) A real symmetric matrix is positive definite if and only if all its eigenvalues are positive
- (2) If the quadratic form  $ax^2 + 2hxy + by^2$  is positive definite, then  $a > 0$ ,  $ab > h^2$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)
- (D) (1) and (3) but not (2)

81. Which of the following matrices is non-derogatory.

- (A)  $\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$
- (B)  $\begin{pmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & -4 \end{pmatrix}$
- (C)  $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$
- (D) None of these

82. Which of the following statements is/are true.

- (1) The sections of an ellipsoid which have their centres on a given line envelope a parabolic cylinder
- (2) All the points on a tangent surface of a curve are parabolic
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

83. What is the torsion of the helix  $x = a \cos \theta$ ,  $y = a \sin \theta$  and  $z = a \theta \tan \alpha$ .

- (A)  $\frac{1}{2a} \sin 2\alpha$
- (B)  $\frac{1}{2a} \cos 2\alpha$
- (C)  $\frac{1}{a} \sin 2\alpha$
- (D) None of these

84. Which of the following statements is/are true.

- (1) Torsion of a geodesic equals  $\sqrt{EG - F^2}$
- (2) The condition for a surface to be developable is that  $LN + M^2 = 0$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

85. Which of the following statements is/are true.

- (1) The surface  $xy = (z - c)^2$  is developable
- (2) The orthogonal trajectories of the rectilinear generators of the tangent surface of a circular helix are the involutes of the given helix
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

86. Which of the following statements is/are true.

- (1) If  $X$  is second countable space and  $A \subset X$  is uncountable, then  $A$  has no accumulation point
- (2) If  $X$  is discrete Lindelof space, then  $X$  is countable
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

87. Which of the following statements is/are true.

- (1) The unitary operators on Hilbert space forms a group
- (2) A Sobolev space is a Hilbert space
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

88. Let  $X$  be a  $T_1$  space and  $A$  be a finite subset of  $X$ . Which of the following statements is /are true.

- (1) Every singleton subset of  $X$  is closed
- (2)  $A$  has at least one accumulation point
- (3)  $A$  is a discrete space
- (A) All (1), (2) and (3) are true
- (B) Only (1) and (3) are true
- (C) Only (1) and (2) are true
- (D) Only (2) and (3) are true

89. Which of the following statements is/are true.

- (1) The set of all polynomials in one variable is dense in  $C[a, b]$  with sup metric
- (2) A linear functional  $f$  on a normed space  $X$  is discontinuous if and only if the null space is nowhere dense in  $X$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

90. Let  $X$  be an infinite dimensional normed space with  $F$  a compact linear operator defined on  $X$ . Which of the following statements is/are true.

- (1) For a polynomial  $p$  in one variable,  $p(F)$  is compact linear operator if  $p(0) = 0$ .
- (2) Every non-zero spectral value of  $F$  is an eigenvalue of  $F$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

91. Which of the following statements is/are true.

- (1) Area is a topological property
- (2) Finite dimensional normed space is reflexive
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

92. Which of the following statements is/are true.

- (1) The distance between any two orthonormal vectors in an inner product space is  $\sqrt{2}$
- (2) Every discrete space is locally compact
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

93. Which of the following statements is/are true.

- (1) Euclidean  $m$ -space is separable
- (2) Open interval  $(0, 1)$  is countably compact set
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

94. What is the rank of the cycle matrix of the graph  $K_4 \cup K_5$ .

- (A) 9
- (B) 11
- (C) 13
- (D) 15

95. Which of the following is not a score sequence of a tournament.

- (A)  $[1, 1, 2, 2]$
- (B)  $[1, 2, 2, 2, 3]$
- (C)  $[0, 1, 1, 4, 4]$
- (D)  $[0, 1, 2, 3, 4]$

96. Which of the following statements is/are true.

- (1) A tree with 18 vertices has 18 edges
- (2) A cycle with 18 vertices has 18 edges
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

97. Which of the following statements is/are true.

- (1) Complexity of  $K_4$  is 4
- (2) The sum of the degrees of a tree with 11 vertices is 20
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

98. Which of the following statements is/are true.

- (1)  $K_3 \vee K_2$  is planar
- (2) The number of edges in a planar graph with 7 vertices is 21
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)

99. Let  $x$  and  $y$  be respectively the size and order of the dodecahedron, and  $z$  be the number of regions of the icosahedron. Then  $x + 2y - 3z$  equals

- (A) 10
- (B) 20
- (C) 30
- (D) None of these

100. Which of the following statements is/are true.

- (1) Every cycle has an even number of edges in common with any bond
- (2) For any graph  $G$ ,  $\kappa(G) \leq \delta(G) \leq \lambda(G)$
- (A) Both (1) and (2)
- (B) Only (1)
- (C) Only (2)
- (D) Neither (1) nor (2)